TABLE I

Side Length Ratio	Characteristic Impedance	Side Length Ratio	Characteristic Impedance
5:4	10.993	7:2	69 687
3:2	20.637	4:1	77.630
5:3	26.441	9:2	84.704
2.1	36.771	5:1	90.981
5:2	49.754	11:2	96.718
3:1	60.528	6:1	101.852

TABLE II

Tabulation of Characteristic Impedance of Square Coaxial Line

Z_0	s	Z_0	s	Z_0	s	Z_0	s	Z_0	s	Z_0	s
0.	1 000	17.	1.407	34.	1 907	51.	2.554	68.	3.402	85.	4.522
1.	1 021	18.	1.432	35.	1.940	52.	2.598	69.	3.460	86.	4.599
2.	1.043	19.	1.457	36.	1.974	53.	2.642	70.	3 519	87.	4.676
3.	1.065	20.	1.483	37.	2.008	54.	2.688	71.	3 578	88	4.755
4.	1.087	21.	1 510	38.	2.043	55.	2.734	72.	3.639	89.	4.836
5.	1.109	22.	1.537	39.	2.078	56.	2 780	73.	3.701	90.	4.918
6.	1.132	23.	1.565	40.	2.114	57	2 827	74.	3 764	91.	5.002
7	1.155	24.	1.594	41.	2.151	58.	2 875	75.	3 828	92.	5.086
8	1.178	25.	1 624	42.	2.188	59.	2.924	76.	3 892	93.	5 172
9	1.202	26.	1.653	43.	2.226	60.	2.974	77.	3.958	94.	5.259
10.	1.225	27	1 684	44.	2.265	61.	3.024	78.	4.025	95.	5.347
11.	1.250	28	1 714	45.	2.304	62.	3.075	79	4 092	96.	5.436
12.	1.279	29.	1.746	46.	2.344	63	3 127	80.	4 161	97.	5.525
13	1 306	30	1.777	47.	2.384	64	3.180	81	4 231	98.	5 616
14.	1.332	31.	1 809	48.	2.426	65	3 234	82.	4 302	99.	5 709
15.	1.357	32	1 841	49.	2.468	66.	3 289	83.	4.374	100	5.805
16.	1.382	33.	1 874	50.	2.511	67.	3 345	84.	4.448	101	5.907

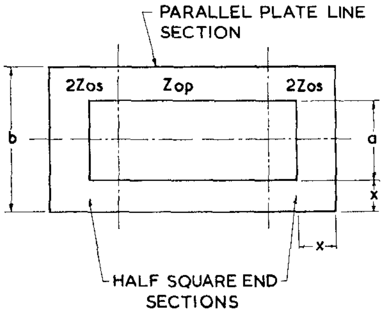

Fig. 2—Development of a square cornered rectangular coaxial line from square section line.

This is presented as Table II. While this introduces the possibility of generating additional errors there is sufficient initial data to suggest that these should remain small. Near the actual data points the basic accuracy of the original work will be preserved. It is pleasing to note the (fortuitous) closeness of the important practical cases of 50, 70 and 75 ohm impedances to basic data points.

When the line is to be filled with a material of dielectric constant K_ϵ the tables should be entered at an impedance of $Z_0\sqrt{K_\epsilon}$, where Z_0 is the required characteristic impedance.

The special case of a rectangular line (Fig. 2) having square corner regions is also covered approximately by this work. Such a line may be considered as the paralleled combination of two half square coaxial lines with a sandwich (parallel plate) line. The accuracy of the approximation will depend on how little the introduction of the parallel plate line disturbs the field at the corners and on what proportion of the capacity these ends contribute. No generalization is therefore possible.

Finally for $s \gtrless 2.0$ it has been found that the data of Table I is fitted with an error not exceeding 0.5 per cent by the empirical relation

$$Z_0 = 136.7 \log_{10} 0.9259s \qquad (2)$$

Acknowledgment

It is a great pleasure to acknowledge the many helpful discussions with B. P. McDowell of Mathematical Services Group whose expert knowledge of digital programming techniques has played a most significant part in this work. The author also wishes to thank the Chief Scientist of the Australian Department of Supply for permission to publish the work.

Harry E. Green
Weapons Research Establishment
Salisbury, South Australia

References

[1] Y. A. Omar and C. F. Miller, "Characteristic impedance of rectangular coaxial transmission lines," *Trans. AIEE*, vol. 71 (*Communication and Electronics*, pt. I), pp. 81–89; January, 1952.
[2] T-S. Chen, "Determination of the capacitance, inductance and characteristic impedance of rectangular lines," IRE Trans on Microwave Theory and Techniques, vol. MTT-8, pp. 510–519; September, 1960.
[3] G. M. Anderson, "The calculation of the electrical capacitance of coaxial cylinders of rectangular cross-section," *Trans. AIEE*, vol. 69, pt. II, pp. 728–731; 1950.
[4] A. Thom and C. J. Apelt, "Field Computations in Engineering and Physics," D. Van Nostrand Co., Inc., New York, N. Y., 1st ed., ch. 4, p. 29; 1961.
[5] R. Culver, "The use of extrapolation techniques with electrical network analogue solutions," *Brit. J. Appl. Phys.*, vol. 3, pp. 376–378; December, 1952.
[6] R. E. Collin, "Field Theory of Guided Waves," McGraw-Hill Book Co., Inc., New York, N. Y., 1st ed., ch. 1, sec. 1, p. 11; 1960
[7] J. J. Skiles and T. J. Higgins, "Determination of the characteristic impedance of U.H.F. coaxial rectangular transmission lines," *Proc. Nat'l. Electronics Conference*, vol. 10, pp. 97–100, 1954.

Postscript to Two Papers on Waffle-Iron Filters*

Introduction

Waffle-iron filters are waveguide "low-pass" filters, that is, they have a wide stop band above their pass band. They are useful for both low-power and high-power applications. A brief history of the development of this filter at Stanford Research Institute is given by Young and Schiffman [1]. A new design principle which makes it possible to extend the pass band from (typically) about 10 per cent to about 40 per cent is also described [1]; the pass band of that filter covers almost the whole of the recommended waveguide band, while its stop band extends from the second to the fourth harmonic, inclusive.

The waffle-iron filter described by Sharp [2] consists of three waffle-iron filters in cascade, to extend the stop band up to the tenth harmonic, inclusive. However, the individual filters had relatively narrow pass bands, and had to be matched to each other by means of quarter-wave transformers.

The purpose of this communication is to show how the wide pass band feature of the one filter [1] was combined with the wide stop band feature of the other filter [2]. At the same time, as the pass band of the original filter [2] was more than doubled, the over-all length of the filter was shortened by almost one half, resulting in an electrically improved as well as a mechanically more compact filter.

The principal design modification made on the original filter [2], was to convert the half-capacitance end sections [2] to half-inductance end sections [1]. This change is simple to make mechanically, but is far-reaching electrically, since the image impedance of the filter now varies with frequency very nearly like the characteristic impedance of the connecting waveguide. This makes it possible to achieve a good match over a very wide pass band. (For a detailed explanation, the reader is referred to Young and Schiffman [1].)

Design Procedure

The original filter consisted of three separate waffle-iron filters designed for separate and slightly overlapping stop bands. They were cascaded, with quarter-wave transformers placed between them to match their different image impedances. The over-all length of the original filter (shown in Fig. 1 of Sharp [2]) was about 30 inches, which was reduced to about 18 inches in the modified filter.

Two design modifications were undertaken:

1) A half section was removed at each end of the three separate filters [2], thus turning the half-capacitance end sections into half-inductance end sections [1], in order to broad band the pass band of each filter.

---

* Received June 17, 1963; revised manuscript received August 7, 1963. This work was supported by the Rome Air Development Center under Contract AF 30(602)-2734.

2) The dimensions of the teeth were then adjusted to make the image impedances of the three separate filters equal to one another, so that no matching transformers would be required between them when they were cascaded.

Since it was proposed to measure the VSWR over a large frequency band, including the whole of L band, it was decided at the outset to design a very good quarter-wave transformer from waveguide height 0.400 inch (the approximate image impedance of the filter [1]) to waveguide height 3.250 inches (WR-650 waveguide). This transformer is described in detail by Young [3]. It was designed for a VSWR of better than 1.01 from 1060 to 1800 Mc (the L-band frequency limits are 1120–1700 Mc).[1]

New Dimensions and Pass-Band Performance of the Three Filters Separately

Fig. 1 shows the cross sections of Waffle-Iron Filter I after a half section was removed from each end, thus reducing the filter from ten to nine sections in length, and (more importantly) changing the ends from half-capacitances [2] to half-inductances [1]. Waffle-Iron Filters II and III are similar in cross section. The final dimensions are given in Table I. The pass band VSWR characteristics of the three separate filters are shown in Fig. 2. The VSWR was measured using the two seven-section quarter-wave transformers to match the input and output from 0.400-inch high to 3.250-inches high (WR-650) waveguide. (The measured VSWR includes the small mismatch of the transformers. Since the seven-section transformer VSWR is less than 1.01, the VSWR of the filters plotted in Fig. 2 and the succeeding figures is essentially the VSWR of each filter between the 6.500 by 0.400 inch waveguides.) The region of most interest is around 1300 Mc, and so Waffle-Iron Filter II requires the most improvement as can be seen from Fig. 2. At this point, instead of changing the tooth dimensions the effective image impedance was changed by adding four posts, as shown in Fig. 3. The posts were 0.156 inch in diameter and were placed between opposing teeth of the two interior rows, close to the waveguide wall (Fig. 3).

Pass-Band Performance of Waffle-Iron Filters in Cascade

The VSWR of Waffle-Iron Filters I and II in cascade is shown in Fig. 4, both with and without the matching posts in Waffle-Iron Filter II. (The stop band of Waffle-Iron Filters I and II in cascade extends from 2.1 to 9.6 Gc [2].) Finally, all three waffle-iron filters were cascaded. A photograph of the composite filter is shown in Fig. 5. The VSWR of this composite filter is shown in Fig. 6. This includes two capacitive bars placed symmetrically in the transformer section that connects to Waffle-Iron Filter III. The reason for the bars is that the image impedance of Waffle-Iron Filter III corresponds to a waveguide height of less than 0.400 inch, as can be seen from Fig. 2; however, instead of modifying Waffle-Iron Filter III any further, or redesigning the transformer, it was quicker to adjust a shunt capacitance to improve the over-all VSWR. The dimensions and position are shown in Fig. 7. It is seen from Fig. 6 that the three waffle-iron filters, cascaded as shown in Fig. 5, then have a VSWR of less than 1.15 from 1190 to 1560 Mc, and remain below 1.4 over the whole of L band (1120–1700 Mc). With a little more effort, modifying the filters and eliminating the posts and bars, the VSWR could probably be improved further, or the pass band extended, or both. Even as it is, however, the performance compares favorably with the earlier filter [2], whose VSWR is also shown in Fig. 6 for comparison, and which had a VSWR of less than 1.15 from 1260 to 1380 Mc only (and remained below 1.4 from 1225 to 1525 Mc only), corresponding to a bandwidth of less than one-half that of the modified filter.

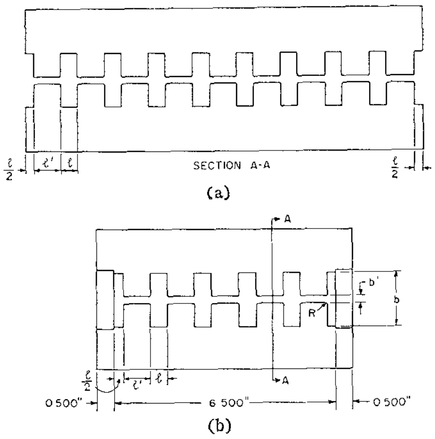

Fig. 1—Cross-sections of a waffle-iron filter. (a) Longitudinal. (b) Transverse.

TABLE I

Final Dimensions of Waffle-Iron Filters

Waffle Iron Filter	Dimensions (inches)				Number of Sections		Length (inches)	Radius of Rounded Edges (inches)
	b''	b	l	l'	In Width	In Length		
I	0.210	1.625	0.560	0.740	5	9	11.700	0.063
II	0.210	0.986	0.231	0.360	11	6	3.546	0.063
III	0.210	0.740	0.230	0.176	16	8	3.248	0.045

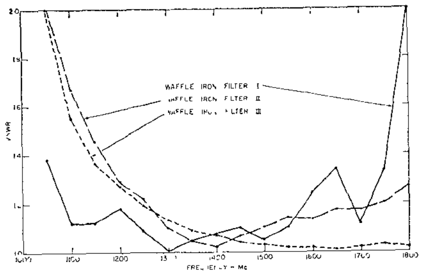

Fig. 2—VSWR of three waffle-iron filters between two 6.500-inch by 0.400-inch waveguides.

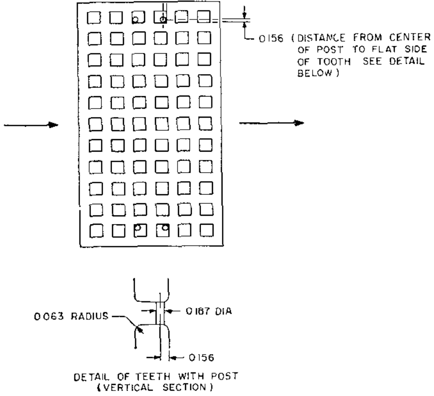

Fig. 3—Waffle-Iron Filter II, showing matching post dimensions.

The three filters are shown assembled in Fig. 5. The over-all length of the filter is 18.494 inches compared with a little over 30 inches for the earlier filter (a reduction of 9.5 inches is due to the intermediate transformers, and about 2 inches is due to the removal of one section per filter).

Stop Band Performance of the Composite Filter

The attenuation in the stop band was measured as before [1], [2]. Drillrods were set in the filter as described by Young and Schiffman [1], to avoid any possible TE$_{01}$-mode resonance (or, more generally, TE$_{m1}$-mode resonance). These can be seen in Fig. 5 as lines running from the upper left to the lower right in the lower half of the photograph. The diameter of the drill rods was 1/16 inch in Waffle-Iron Filters I and II, and 3/64 inch in Waffle-Iron Filter III. (Without the drill rods there are about a dozen sharp spikes where the attenuation falls to between 35 and 45 db.) The attenuation measurements were made as described before [1], [2], and were limited by the sensitivity of the equipment. One would expect the modified filter to have less attenuation than the original filter for two reasons: First, the number of sections has been reduced by one in each of the three component filters (*e.g.*, from ten to nine in Waffle-Iron Filter I). Second, the change in dimensions (compare Table I with Table I of Sharp [2]) is in such a direction as to slightly increase the cutoff frequency of Filters I and III.

Experimentally, no deterioration in the stop band was observed compared with the previously published curves [2]. The attenuation was at least 60 db from 2.1 to 13.8 Gc, except in two regions: the sensitivity fell from 60 db to 55 db between 6.8 to 7.5 Gc, so that it is not known for certain whether it exceeded 60 db in this region; and the attenuation dropped measurably, but remained better than 55 db in the region from 9.7 to 10.3 Gc. With these exceptions,

---

[1] This transformer is essentially a piece of laboratory test equipment and a shorter transformer of five sections (instead of seven) would give an adequate match over the pass-band region. For Waffle-Iron Filters I, II, and III in cascade, the pass band was later determined to be 1190 to 1560 Mc for a VSWR of less than 1.15; or the whole of L band may be considered the pass band with a VSWR of less than 1.4.

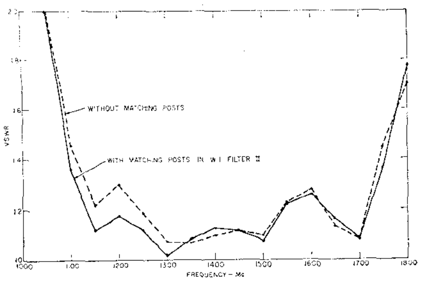

Fig. 4—VSWR of Waffle-Iron Filters I and II in cascade between two 6.500-inch by 0.400-inch waveguides.

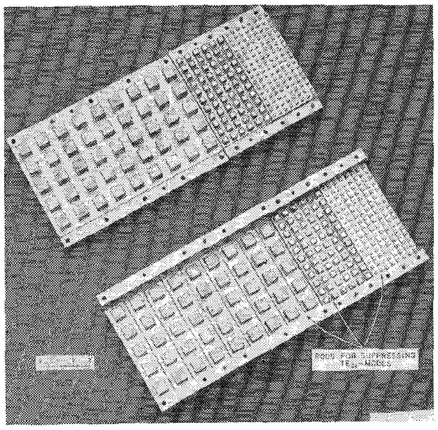

Fig. 5—Exploded view of three waffle-iron filters in cascade.

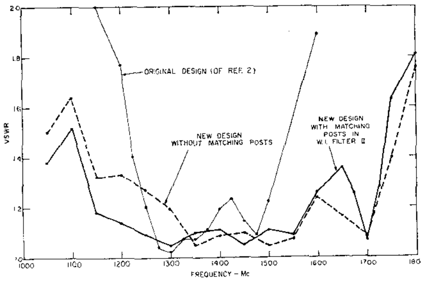

Fig. 6—VSWR of Waffle-Iron Filters, I, II, and III in cascade between two 6,500-inch by 0.400-inch waveguides, with capacitive bars near Waffle-Iron Filter III. (The VSWR of the original design [2] is also shown.)

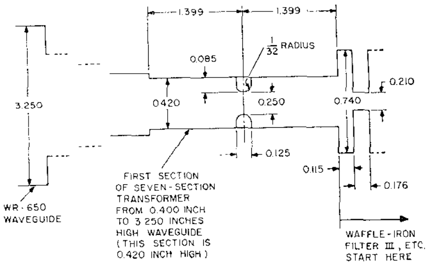

Fig. 7—Dimensions and positions of capacitive bars near Waffle-Iron Filter III.

the attenuation remained above 60 db from the second to the tenth harmonic of 1.3 Gc. It fell below 60 db above 13.8 Gc, but remained above 45 db to 14.5 Gc, after which the attenuation fell off rapidly. These results are consistent with Fig. 13 of Sharp [2]. To ensure 60 db of stop band attenuation, the number of sections in Waffle-Iron Filter II should probably be increased from 6 to 7 or 8. The wide stop band of the waffle-iron filter makes it useful in suppressing harmonic and other spurious frequencies, whether from laboratory signal generators or high-power transmitters [4].

ACKNOWLEDGMENT

The help and cooperation of B. M. Schiffman are very much appreciated. The experimental measurements were ably made by R. Larrick.

LEO YOUNG
Stanford Research Institute
Menlo Park, Calif.

REFERENCES

[1] L. Young and B. M. Schiffman, "New and improved types of waffle-iron filters," *Proc. IEE (London)*, vol. 110, pp. 1191–1198; July, 1963.
[2] E. Sharp, "A high-power wide-band waffle-iron filter," IEEE TRANS. ON MICROWAVE THEORY AND TECHNIQUES, vol. MTT-11, pp. 111–116; March, 1963.
[3] L. Young, "Practical design of a wide-band quarter-wave transformer in waveguide," *Microwave J.*, vol. 6, pp. 76–79; October, 1963.
[4] L. Young, B. M. Schiffman, and O. Allen, "The Waffle-Iron Filter—A Microwave Filter to Suppress Spurious Frequencies at the Source," presented at the 5th Nat'l. Symposium on Radio Frequency Interference, Digest Sec. III-B, Philadelphia, Pa.; June, 1963.

The Effect of Increased Design Bandwidth Upon Direct-Coupled-Resonator Filters*

INTRODUCTION

The Cohn synthesis[1] is generally preferred as a convenient design technique for direct-coupled waveguide filters covering a moderate bandwidth. Approximations inherent in the theory result in considerable deviations from the theoretical pass band response as the design bandwidth increases. The magnitude of the deviations is difficult to predict and proves troublesome in designs requiring a low pass band VSWR.

Some idea of the deviations likely to occur in practice has been obtained with the aid of a computer for the particular case of a 7-cavity filter. The method employed was to use the Cohn synthesis to calculate the filter parameters necessary to satisfy a particular specification. The resultant filter structure of shunt inductance elements, separated by lengths of transmission line, can be represented conveniently by matrices of rank two. In this representation the frequency response of such a structure can be swiftly evaluated with the aid of a conventional digital computer.

Thus the exact response of the synthesized filter structure can be found and the deviation from the theoretical curve determined.

EVALUATION PROCEDURE

In practice the susceptance of a post or iris in waveguide is given by a complicated function of guide wavelength, which is difficult to fit into a computer program. To avoid narrowing the scope of the analysis a simple proportionality relationship was assumed between the susceptance and guide wavelength. The other approximation in the analysis was the neglect of absorption in the waveguide. This assumption was justified for the bandwidths of interest in this investigation. Pass band responses were restricted to the equal-ripple Chebyshev type, and were computed for a range of bandwidths from 1 per cent to 20 per cent, centered at 9.5 kMc, in wg 16.

Results for specified pass band VSWR ripples of 1.05, 1.1 and 1.25 are shown in Figs. 1–3, respectively. For convenience all design bandwidths have been normalized in width on a frequency scale, so that the design equal-ripple points on the slopes of the characteristic coincide. The change in pass band response with increase in bandwidth, or reduction of the specified pass band VSWR, is thus made readily apparent. Two main effects can be observed from the curves,

1) For the responses in Figs. 1 and 2 large peaks occur near each end of the pass band. These peaks grow in amplitude with increase in design bandwidth, the upper peak shifting towards the center of the pass band. There is no sign of such peaks in Fig. 3.
2) A shift down in frequency of the upper skirt of the filter response occurs in all cases. This shift increases with increase in design bandwidth. There is a much smaller shift upwards in frequency of the lower skirt.

The difference in effect upon the upper and lower slopes of the response and the shift in the upper frequency VSWR peak is consistent with the symmetry of the design pass band response on a guide wavelength scale. This results in the design center frequency shifting towards the low-frequency end of the pass band. Thus any approximations in the synthesis which are dependent upon frequency will be more evident at the high-frequency end of the pass band.

Computations were made with higher values for the specified pass band VSWR. The results obtained were similar to those shown in Fig. 3. In general there is a slight decrease in the amplitude of the VSWR ripples, as the design bandwidth increases, accompanied by comparable shift in the attenuation slopes of the response.

ERROR IN COMPUTED CURVES

An attempt was made to estimate the error in the computed curves resulting from the approximate relation assumed for the guide wavelength. Because of a practical requirement measurements were made upon a 9-cavity wg 16 filter. The filter was manufactured using single posts of circular cross section for the inductive susceptances. Dimensions and positions for the posts were obtained from computations based upon formulas determined by Lewin.[2] These

---

* Received August 12, 1963.

[1] S. B. Cohn, "Direct-coupled-resonator filters," PROC. IRE, vol. 45, pp. 187–196; February, 1957.

[2] L. Lewin, "Advanced Theory of Waveguides," Iliffe and Sons, Ltd., London, England, p. 35; 1951.